Vibrational Characteristics of Composite Panels with Cutouts

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A method is presented for analyzing the free damped vibration characteristics of plates, cylinders, and cylindrical panels made of fiber reinforced, organic matrix composites. A computer code is described that can be used to calculate the natural frequencies, mode shapes, and damping factors of plates, cylinders, and cylindrical panels with free, clamped, or simply supported edges and with or without circular cutouts. Tests also were performed measuring the damping factors of Fiberite T300/934 graphite-epoxy unidirectional composite and the natural frequencies and mode shapes of rectangular plates made of this material. Plates with $[0_8]$, $[(45/-45)_2]_s$, and $[(0/90)_2]_s$ layups were tested. Measurements were made with "solid" plates and with plates containing centrally located circular cutouts. The measured and calculated natural frequencies and damping factors were compared. The data and the numerical results were found to be in good agreement. As an illustrative example, the natural frequencies, damping factors, and mode shapes also were calculated for a semicircular cylindrical panel made of Fiberite T300/934 graphite-epoxy unidirectional tape.

Introduction

In recent years, composites have gained widespread acceptance as panels, coverings, and skins in aircraft, spacecraft, and ground vehicles where the light weight and high stiffness of the material is of great advantage. However, such structural elements tend to vibrate. Therefore, to use composites as structural members, their vibration characteristics must be understood, and methodologies for the design of composite structures must be available.

Recognizing the significance of the problem, many investigators have studied the vibration of composite plates and shells. Most previous investigations were concerned with the free undamped vibration of composites. Less attention has been paid to the problem of free damped vibration of composites, with most of the previous studies addressing the issue of material damping.[1-14] There are relatively few analytical investigations on the problem of free damped vibration of composite plates and shells. Lin et al.[15] and Alam and Asnani[16] presented analyses of the free vibration of composite plates. Alam and Asnani[17-19] also studied the free damped vibration of circular cylinders made of either alternating layers of different isotropic materials or a specially orthotropic composite. It appears that in-depth results have not been reported yet for the problem of free damped vibration of curved composite panels and composite panels containing cutouts.

The importance of the problem, combined with the paucity of information, led to our investigation of the free damped vibration of composite plates, cylinders, and cylindrical panels containing circular cutouts.

Problem Statement

We consider flat plates, cylinders, and cylindrical panels (Fig. 1). There may be either one or two circular holes (cutouts) of radius r in these structural elements. In the case of a single cutout, it is located centrally. In the case of two cutouts, they are located symmetrically about a line of symmetry.

Each outer edge may be clamped, simply supported, or free. No other forces or constraints are applied.

The plate, cylinder, or panel is made of a laminated composite that consists of layers of unidirectional, continuous fibers embedded in an organic matrix. Perfect bonding between each layer is assumed. The laminates may be made entirely of composite or of two composite facesheets enclosing a core, but the cross section must be symmetric with respect to the midsurface of the shell. The thickness is constant and is small compared to all other dimensions.

The composite material used in the construction is assumed to behave in a linearly elastic manner. It also is assumed that damping is light (i.e., any vibration of the material dies out in an amount of time that is large in comparison to the period of vibration) and that damping of the material is independent of the frequency of the vibration. This latter assumption, although generally invalid for an isotropic metal, is often justified for fiber-reinforced organic matrix composites.[9,10] The vibrations are taken to be simple harmonic motions, with the effects of gravity on the vibration considered to be negligible.

It is desired to determine the natural frequencies, mode shapes, and damping factors of the composite plates, cylinders, and cylindrical panels described above.

Method of Solution

The governing equations and the method of solution were described in detail elsewhere[20] and are not repeated here. Only

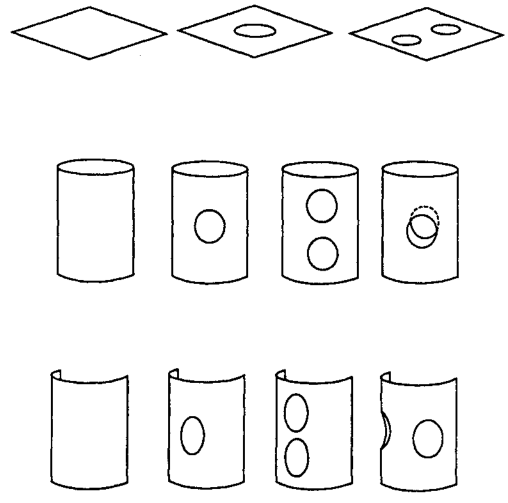

Fig. 1 Geometries of the plates, cylinders, and cylindrical panels investigated.

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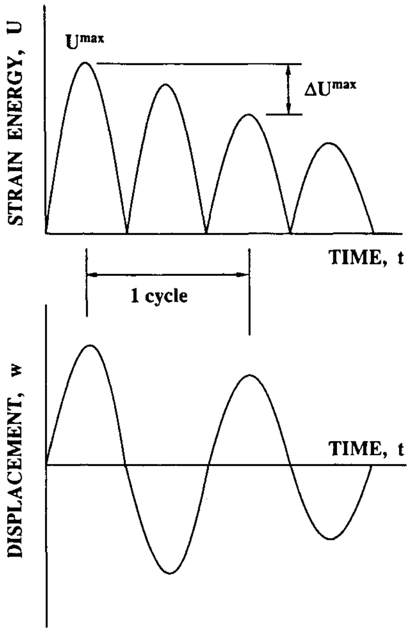

Fig. 2 Illustration of the terms used in the definition of the damping factor.

a brief summary is given, which is needed in understanding the use of the model and its limitations.

The equations governing the vibration of the structure were derived along the lines proposed by Reddy,[21] using both a first-order and a higher-order shear deformation approximation. Numerical values for the natural frequencies, mode shapes, and damping factors were generated by a finite-element method. The problem was attacked in two steps. In the first step, the undamped natural frequencies and the corresponding undamped mode shapes of the shell were obtained. For lightly damped structures, such as those considered in this study, the undamped natural frequencies and mode shapes are nearly the same as the damped natural frequencies and mode shapes. In the second step, the damping corresponding to each mode of vibration, as expressed in terms of a modal damping factor, was calculated.

Damping

Damping was included in the analysis as follows. The damping associated with a given mode of vibration of the shell is characterized by a modal damping factor η (also called the loss factor) defined as[22]

$$\eta = \frac{\Delta U^{\max}}{2\pi U^{\max}} \tag{1}$$

where η is a measure of the strain energy dissipated per radian of vibration in the mode of interest, $U^{\max}$ the total strain energy of the entire laminate at maximum displacement during one cycle of vibration in the mode of interest, and $\Delta U^{\max}$ the strain energy dissipated in that mode of vibration during the same cycle (Fig. 2).

Proceeding in a manner similar to Lin et al.,[15] for a laminate consisting of N plies, we write

$$U^{\max} = \sum_{n=1}^{N} U_n^{\max} \tag{2}$$

where $U_n^{\max}$ is the strain energy of the nth ply at the maximum displacement during one cycle of vibration in the mode of interest. Similarly, the strain energy dissipated by the laminate during the same cycle of vibration in the mode of interest is

$$\Delta U^{\max} = \sum_{n=1}^{N} \Delta U_n^{\max} \tag{3}$$

where $\Delta U_n^{\max}$ is the strain energy dissipated by the nth ply during this cycle of vibration in the mode of interest. The strain energy for an orthotropic ply, in terms of the ply coordinates, is[20]

$$U_n = \int_{V_n} (\tfrac{1}{2} Q_{xx} e_x^2 + \tfrac{1}{2} Q_{yy} e_y^2 + Q_{xy} e_x e_y + 2Q_{yz} e_{yz}^2 + 2Q_{xz} e_{xz}^2 + 2Q_{ss} e_{xy}^2)_n \, \mathrm{d}V_n \tag{4}$$

where Q_{xx}, Q_{yy}, Q_{xy}, Q_{yz}, Q_{xz}, and Q_{ss} are the components of the reduced stiffness matrix[23]; e_x, e_y, e_{xy}, e_{xz}, and e_{yz} are the strain components; and V_n is the ply volume. The coordinates x and y are in the directions parallel and transverse to the fibers in the plane of the ply, and z is in the direction normal to the plane of the ply. The subscript ss represents in-plane shear. From Eq. (4), the maximum strain energy during one cycle of vibration can be written as

$$U_n^{\max} = (U_1 + U_2 + U_3 + U_4 + U_5 + U_6)_n^{\max} = \sum_{i=1}^{6} (U_i)_n^{\max} \tag{5}$$

Analogous to Eq. (5), the loss of strain energy of the nth orthotropic ply is written as

$$\Delta U_n^{\max} = (\Delta U_1 + \Delta U_2 + \Delta U_3 + \Delta U_4 + \Delta U_5 + \Delta U_6)_n^{\max} = \sum_{i=1}^{6} (\Delta U_i)_n^{\max} \tag{6}$$

Each of the six terms in Eq. (6) represents the change in the strain energy associated with U_1 through U_6 during one cycle. Analogous to Eq. (1), for the nth ply a damping factor now is defined for each of the six strain energy terms

$$(\eta_i)_n = \left(\frac{\Delta U_i^{\max}}{2\pi U_i^{\max}} \right)_n, \qquad i = 1,2,\ldots,6 \tag{7}$$

where $U_i^{\max}$ is the strain energy at maximum displacement and $\Delta U_i^{\max}$ the corresponding strain energy dissipated during the ensuing one cycle of vibration.

By combining Eqs. (1–7) we obtain for each vibration mode of interest the modal damping factor for the laminate

$$\eta = \frac{\sum_{n=1}^{N} \sum_{i=1}^{6} (\eta_i U_i)_n^{\max}}{\sum_{n=1}^{N} \sum_{i=1}^{6} (U_i)_n^{\max}} \tag{8}$$

For each mode of vibration, the strains needed to calculate the strain energies are given by the free undamped vibration of the equations of motion.[20]

Finite-Element Method

A finite-element procedure was developed for obtaining solutions to the problem of free damped vibrations of composite plates, cylinders, and cylindrical panels with or without circular cutouts. The finite-element method was formulated using the first-order shear deformation theory because, for the problems considered in this study, this approximation was found to provide results of sufficient accuracy.

In the finite-element method we employed four-node bilinear quadrilateral elements with five degrees of freedom per node. To minimize errors in plates (caused by shear locking), the entries in the element stiffness matrix associated with transverse shear stiffnesses were evaluated using the one-point Gaussian integration scheme. To minimize errors in shells (caused by shear and membrane locking), the entries in the element stiffness matrix associated with transverse shear and membrane stiffnesses were evaluated using the one-point Gaussina integration scheme (selective reduced integration, SRI, or $\bar{B}$-method[24]). For further details of the finite-element formulation the reader is referred to Ref. 25.

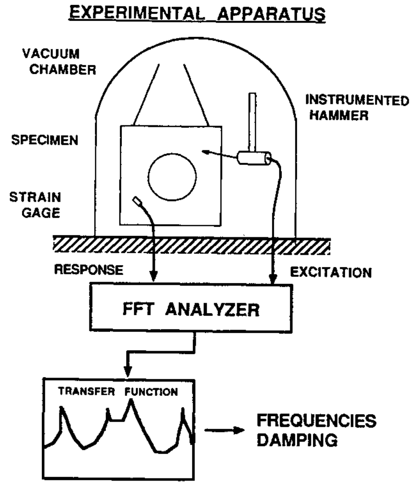

Fig. 3 Schematic of vibration test setup.

We developed a user-friendly computer code (designated as "VIBR8") to implement the algorithm. This code (which may be obtained from the authors) can be used to calculate the natural frequencies, mode shapes, and damping factors of composite plates, cylinders, and cylindrical panels with and without circular cutouts.

An assessment of the accuracy of the algorithm and computer code is given subsequently. First, in the next section, the experiments are described that were performed to generate the data against which the numerical results were compared.

Experiments

Two types of tests were performed. First and foremost, tests were conducted to assess the accuracy and validity of the analytical model. Second, tests were conducted to measure the material properties that were required in the analysis but were not available in the literature.

Test Specimens

The test specimens were cut from 6 by 6 in. plates made from Fiberite T300/934 graphite-epoxy unidirectional tape. Plates of three different layups were used in the tests to verify the model: $[0_8]$, $[(0/90)_2]_s$, and $[(45/-45)_2]_s$. All these plates were tested without any cutouts and with a single hole drilled in the middle of the plate. The holes were 1 in., 2¼ in., and 3¼ in. in diameter.

Plates consisting of eight plies also were made with the fibers in each of the eight plies all running parallel to each other. These plates were cut into 6 by 1 in. beams. In five of these "unidirectional" beams, the fibers were running along the length of the beam; in five, the fibers were running transverse to the length of the beam.

In addition, 0.07 in. thick plates made of Fiberite 934 epoxy resin were fabricated. These plates were made by pouring the rein into a mold and by curing the plates. We cut 6 by 1 in. beams from these plates.

A strain gage was mounted on each specimen (beam or plate). The strain gage was located as far as possible from the nodal lines. The nodal lines were determined from the analytical model.

The 6 by 6 in. composite plates were used to measure the natural frequencies and damping factors. The unidirectional composite and pure resin beam specimens were used to measure the damping factors of the unidirectional material.

Experimental Apparatus and Procedure

In order to obtain the natural frequencies and the damping factors of the beams and plates, henceforth referred to as the specimens, the specimen was suspended by thin $(4 \times 10^{-3}$ in. diameter) wires inside a vacuum chamber (Fig. 3). The wires were attached to the specimen through small holes located along the nodal lines. The nodal lines were estimated prior to the tests by the model. The pressure inside the vacuum chamber was kept below 2×10^{-3} mm Hg. The tests were performed at low pressure to minimize damping due to air.

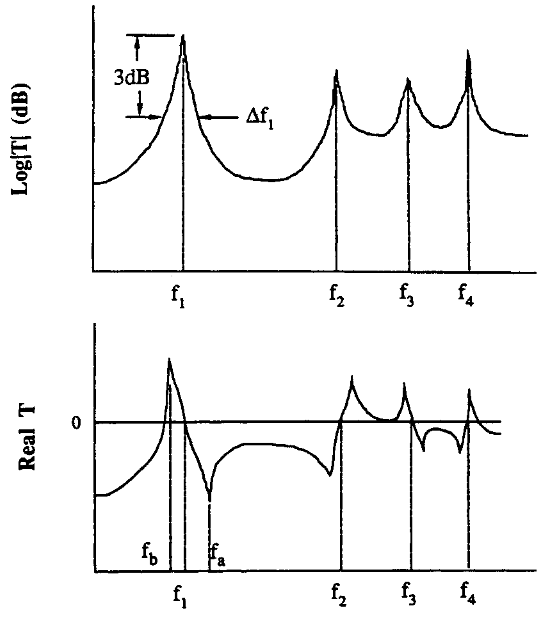

Fig. 4 Illustration of the output of the structural dynamics analyzer. Top: the log of the magnitude of the transfer function T as a function of frequency. Bottom: the real part of the transfer function T as a function of frequency. f_1, f_2, f_3, and f_4 are the natural frequencies.

Each specimen was struck by a hammer at various locations. The hammer contained a force transducer, which measured the force of the hammer's impact on the specimen. The output from the force transducer was fed into a HP-5423A Structural Dynamics Analyzer. The output of the strain gage also was fed into this analyzer. The analyzer provided directly the transfer function, which is the ratio of the Fourier transform of the strain gage response to the Fourier transform of the hammer excitation.

A typical output from the structural dynamics analyzer is illustrated in Fig. 4. The upper plot shows the magnitude of the transfer function, plotted on a logarithmic scale vs the frequency. The lower plot shows the real part of the transfer function vs the frequency. Plots such as these were generated for each specimen. The natural frequencies (resonances) for each mode $(f_1, f_2, f_3, \ldots)$ correspond to the peaks of the transfer function vs the frequency curve (Fig. 4, top).

The damping factors were determined in two different ways. In the first method the following equation was used[26]

$$\eta_r = [(\Delta f)_r / f_r] \qquad r = 1,2,3,\ldots \tag{9}$$

where η_r is the damping factor of mode r, $(\Delta f)_r$ the frequency interval between two points that are 3 dB below the resonant peak associated with mode r on the transfer function vs the frequency curve (Fig. 4, top), and f_r the natural frequency for mode r.

The second method employed the following equation[26]

$$\eta_r = \frac{(f_a/f_b)_r^2 - 1}{(f_a/f_b)_r^2 + 1} \qquad r = 1,2,3,\ldots \tag{10}$$

where f_a is the frequency corresponding to the peak (maximum or minimum) of the real part of the transfer function *above* the resonance frequency of mode r and f_b the frequency corresponding to the peak of opposite sign of the real part of the transfer function *below* the resonance frequency of mode r (Fig. 4, bottom).

The first method tended to give higher damping factors than the second method. The damping factors presented in Table 1 and in the next section are the algebraic averages of the values obtained by Eqs. (9) and (10).

Using the above experimental procedure, two sets of data were generated. First, we measured the damping factors of 934 epoxy resin and Fiberite T300/934 unidirectional material (Table 1). Second, we measured the natural frequencies and damping factors of the plate specimens. The data for the plates are presented in detail in the next section.

Results

Since the major focus of the present study was on the free damped vibration of panels containing circular cutouts, it was felt imperative to validate the present method used for these specific problems. Since no analytical and experimental results were available, it was necessary to generate our own data, which then could be used to test the model and the computer code. Therefore, we performed tests measuring the natural frequencies and damping factors of 6 in. square plates made of Fiberite T300/934 graphite-epoxy unidirectional tapes. Plates with $[0_8]$, $[(45/-45)_2]_s$, and $[(0/90)_2]_s$ layups were tested as described in the previous section. The test were performed with "solid" plates and with plates containing centrally located circular cutouts.

The data are presented in Figs. 5-10. Each data point in these figures is the average of 5 to 15 measurements. The standard deviations in the data are represented by the error bars. When no error bars are shown, the standard deviation was less than the size of the data point.

Table 1 Measured Damping Factors of T300/934[a]

Damping factor		Standard deviation
Longitudinal	$\eta_x = 0.0076$	0.0004
Transverse	$\eta_y = 0.0180$	0.0027
Shear	$\eta_s = 0.0235$	0.0021
934 epoxy	$\eta_m = 0.0490$	0.0092

[a]Fiber volume fraction $v_f = 0.8$.

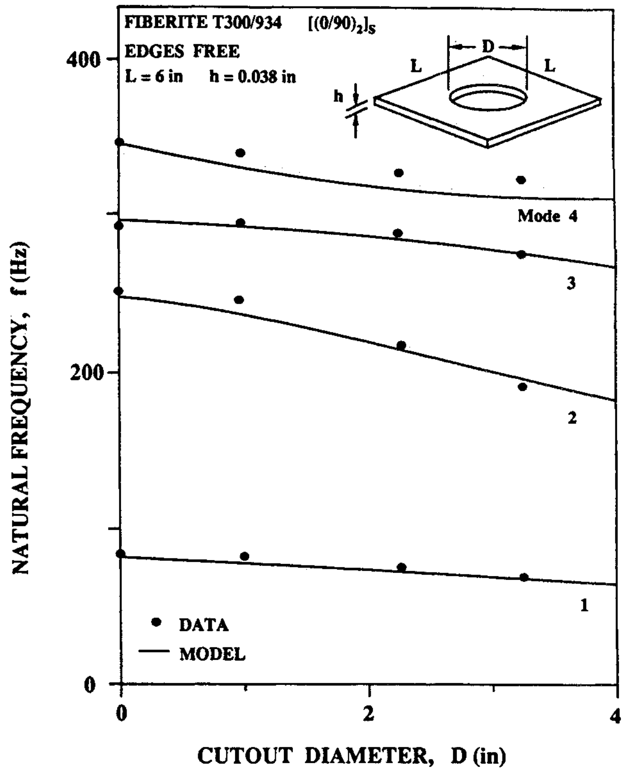

Fig. 6 Natural frequencies as a function of cutout diameter of a $[(0/90)_2]_s$ graphite-epoxy square plate with fiber volume fraction v_f of 0.76. Comparison of the results of the present method with the data generated in this study. Material properties given in Tables 1 and 2.

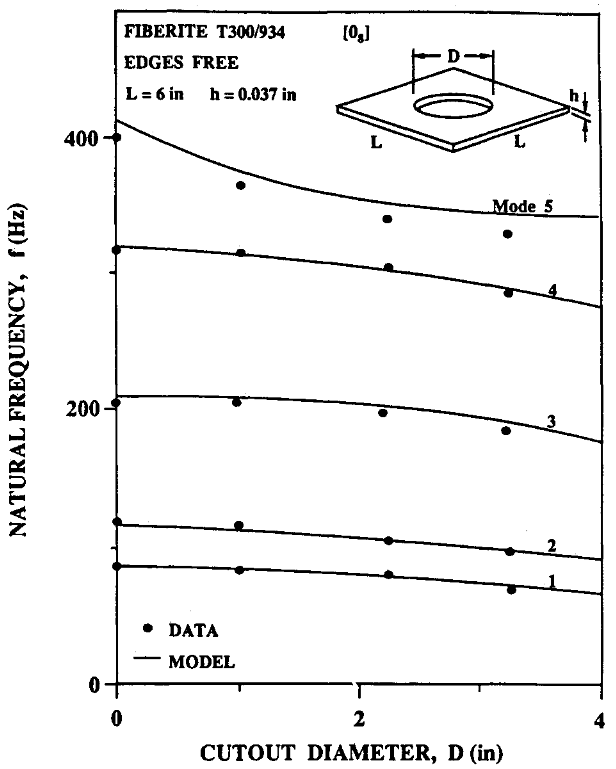

Fig. 5 Natural frequencies as a function of cutout diameter of a graphite-epoxy square plate made of eight plies of unidirectional tape with fiber volume fraction v_f of 0.8. Comparison of the results of the present method with the data generated in this study. Material properties given in Tables 1 and 2.

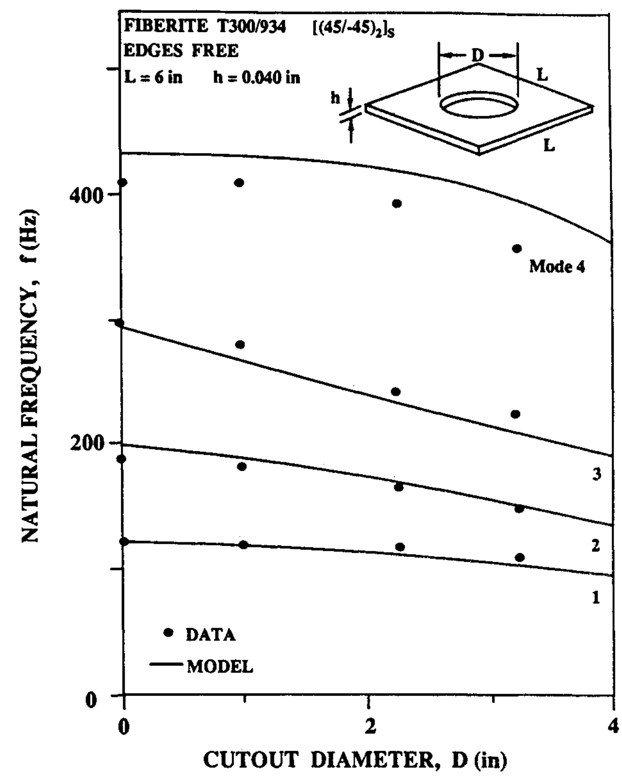

Fig. 7 Natural frequencies as a function of cutout diameter of a $[(45/-45)_2]_s$ graphite-epoxy square plate with fiber volume fraction v_f of 0.7. Comparison of the results of the present method with the data generated in this study. Material properties given in Tables 1 and 2.

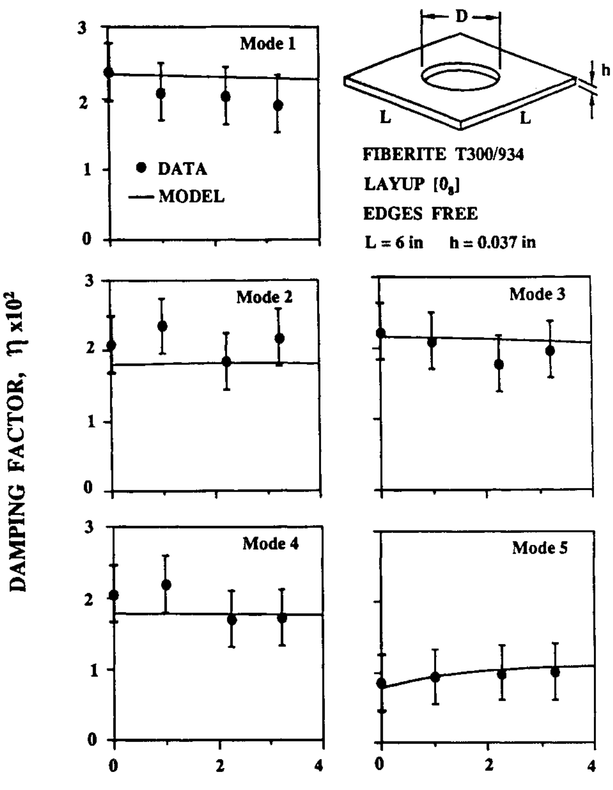

Fig. 8 Damping factors as a function of cutout diameter of a graphite-epoxy square plate made of eight plies of unidirectional tape with fiber volume fraction v_f of 0.8. Comparison of the results of the present method with the data generated in this study. Bars represent the standard deviation of the data. Material properties given in Tables 1 and 2.

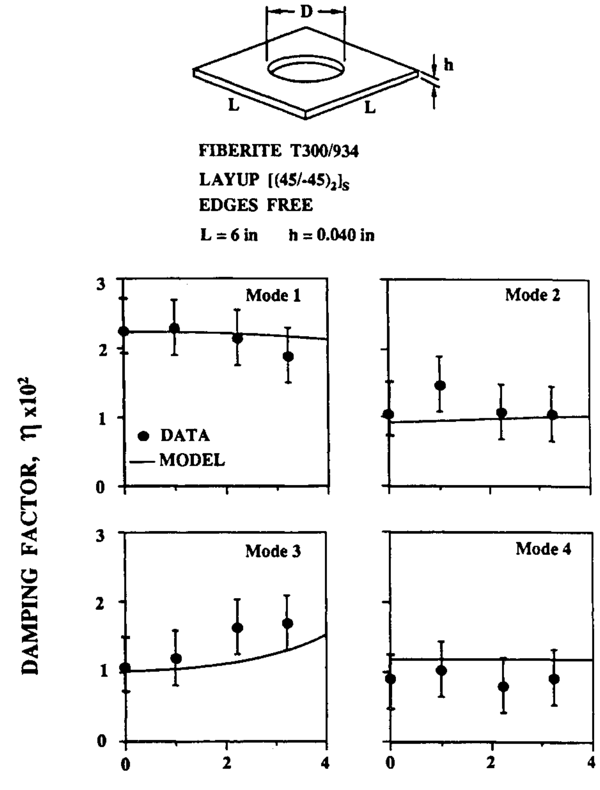

Fig. 10 Damping factors as a function of cutout diameter of a $[(45/-45)_2]_s$ graphite-epoxy square plate with fiber volume fraction v_f of 0.7. Comparison of the results of the present method with the data generated in this study. Bars represent the standard deviation of the data. Material properties given in Tables 1 and 2.

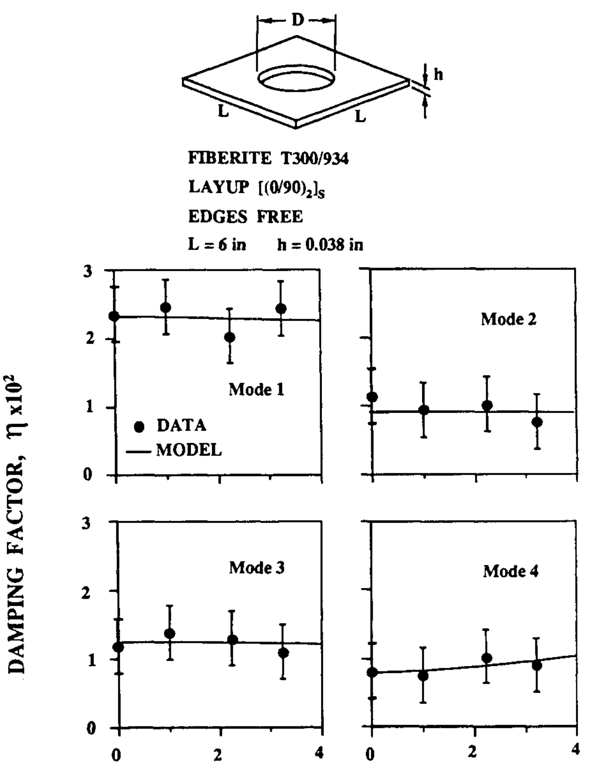

Fig. 9 Damping factors as a function of cutout diameter of a $[(0/90)_2]_s$ graphite-epoxy square plate with fiber volume fraction v_f of 0.76. Comparison of the results of the present method with the data generated in this study. Bars represent the standard deviation of the data. Material properties given in Tables 1 and 2.

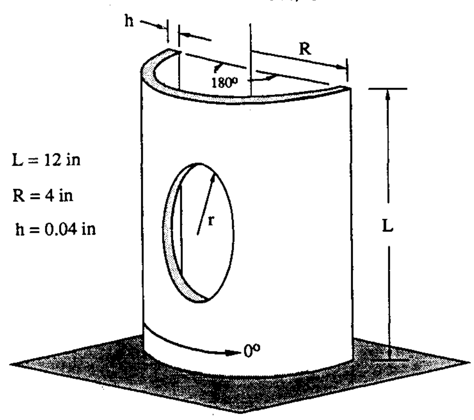

Fig. 11 Description of the semicircular cylindrical panel used in the sample problem.

The natural frequencies calculated by the present method are compared with the data in Figs. 5-7. The comparison between the damping factors calculated by the present method and the data are presented in Figs. 8-10. As these figures shown, there is good agreement between the results of the present method and the data. This gives us confidence that our model and computer code can be used to analyze the free damped vibration of plates and shells containing circular cutouts and to predict the natural frequencies, mode shapes, and damping factors with reasonable accuracy.

Sample Problem

In order to illustrate the type of information that can be obtained by the "VIBR8" computer code, results were gener-

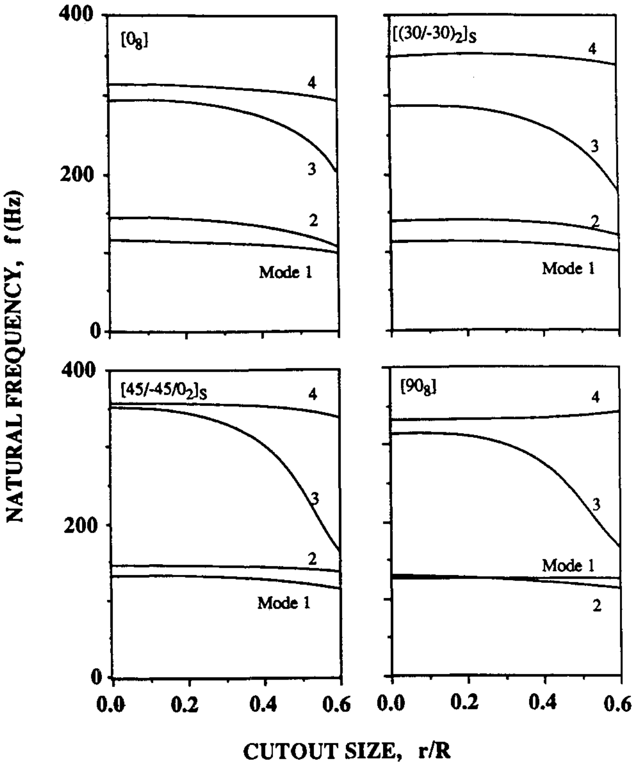

Fig. 12 Natural frequencies as a function of the cutout size of the graphite-epoxy semicircular cylindrical panel shown in Fig. 11, with four different ply layups: $[0_8]$, $[(30/-30)_2]_s$, $[45/-45/0_2]_s$, and $[90_8]$. Direction of 0 deg plies shown in Fig. 11. Material properties given in Table 1 and 2.

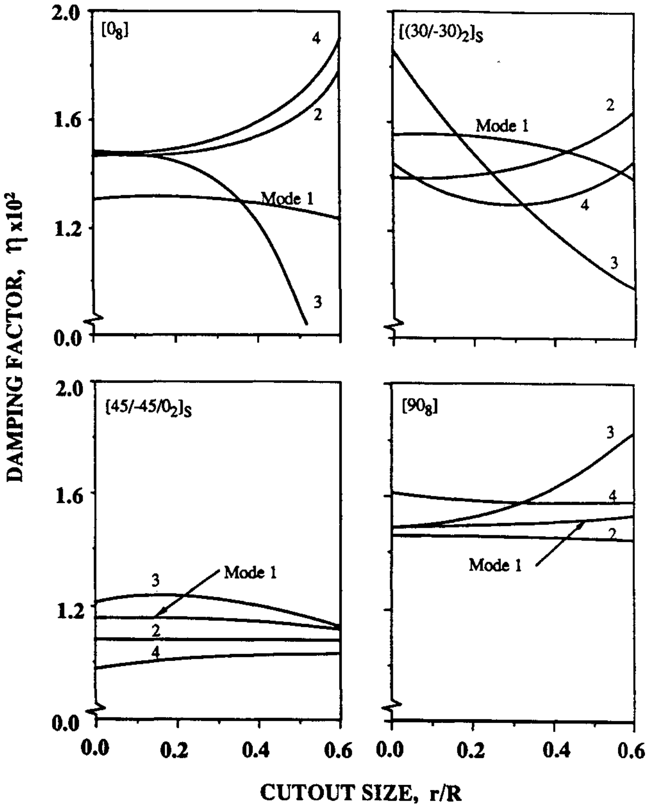

Fig. 13 Damping factors as a function of the cutout size of the graphite-epoxy semicircular cylindrical panel shown in Fig. 11, with four different ply layups: $[0_8]$, $[(30/-30)_2]_s$, $[45/-45/0_2]_s$, and $[90_8]$. Direction of 0 deg plies shown in Fig. 11. Material properties given in Table 1 and 2.

Table 2 Material properties used in the calculations[a]

Property	Units	T300 fibers	934 epoxy
Longitudinal modulus	Msi	32	0.5
Transverse modulus	Msi	2	
Longitudinal shear modulus	Msi	0.27	1.3
Transverse shear modulus	Msi	1.3	
Transverse shear modulus	Msi	0.7	
Longitudinal Poisson's ratio	—	0.2	0.35
Density	10^{-3} lb-s²/in.⁴	0.166	0.114

[a]Material properties of T300/934 graphite-epoxy composite were calculated using a rule of mixtures.

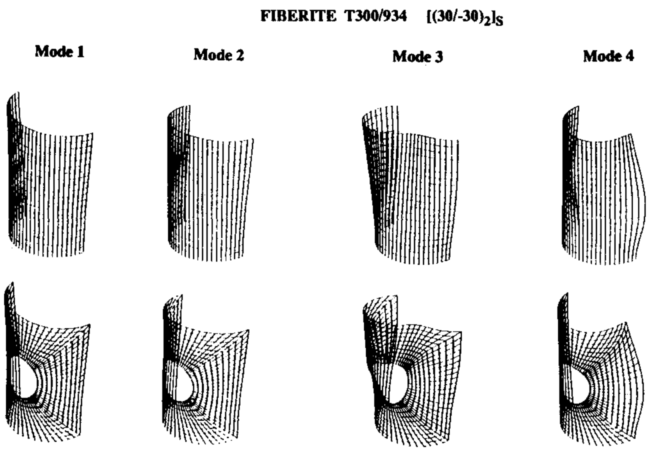

Fig. 14 The first four mode shapes of a $[(30/-30)_2]_s$, semicircular cylindrical panel shown in Fig. 11. Mode shapes are for panels without a cutout (top) and containing a 2 in. radius circular cutout (bottom).

ated for a sample problem. In this problem, the free damped vibrational characteristics of a 12 in. long and 8 in. diameter semicircular cylindrical panel were determined (Fig. 11). One edge of the panel was clamped and the other edges were free. The panel was assumed to be made of Fiberite T300/934 graphite-epoxy unidirectional tape with a fiber volume fraction of 0.5 (Tables 1 and 2). The walls of the panel consisted of eight plies, for a total thickness of 0.04 in. Panels with four different laminate ply orientations were considered: $[0_8]$, $[(30/-30)_2]_s$, $[45/-45/0_2]_s$, and $[90_8]$. Results were generated for panels without cutouts and for panels containing a circular cutout located at the center of the panel. The diameter of the cutout was varied from 0 to 5 in. The calculations were performed using an 800 element mesh. The results are presented in Figs. 12–15.

The first four natural frequencies for different size cutouts are presented in Fig. 12. Generally, the natural frequencies decrease with increasing cutout radius. The largest decrease corresponds to mode 3. Interestingly, for the $[90_8]$ laminate the natural frequencies increase slightly with increasing cutout size for modes 1 and 4.

The damping factors are presented in Fig. 13. This figure shows the important result that, depending on the layup, the damping factors increase or decrease with an increase in cutout radius. Thus, damping may be altered significantly by cutouts. For example, in the case of the $[0_8]$ laminate, the damping factors associated with modes 2 and 4 increase by more than 10% when a cutout of 4 in. diameter ($r/R = 0.5$) is cut in the panel, and the damping factor for mode 3 decreases by more than 50% for the same cutout. The largest change in damping for each of the laminates considered corresponds to mode 3, which is the same mode that has the largest frequency change for each laminate.

The effect of the cutout on the mode shape is demonstrated by the results in Figs. 14 and 15. These figures show the first four mode shapes of the $[(30/-30)_2]_s$ and $[90_8]$ laminates

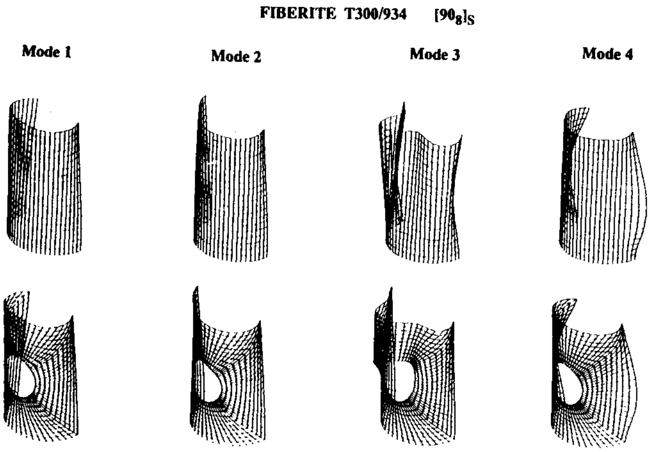

Fig. 15 The first four mode shapes of a $[90_8]$ semicircular cylindrical panel shown in Fig. 11. Mode shapes are for panels without a cutout (top) and containing a 2 in. radius circular cutout (bottom).

discussed above. As shown by these figures, the cutout causes a distortion of the mode shape, especially for mode 3.

The foregoing illustrates the kinds of information that can be generated by the results of the "VIBR8" code. The code can be used as a tool for designing plates, cylinders, and cylindrical panels. In design, numerical results are obtained for different ply orientations, cutout sizes, and material properties. The ply orientation, geometry, and material then are chosen that result in the structure having the desired natural frequencies, mode shapes, and damping factors.

Concluding Remarks

The computer code developed during the course of this study is applicable to problems of free damped and undamped vibrations of "solid" composite flat and cylindrical panels, as well as to panels containing circular cutouts.

The code should not be applied to free *damped* vibration of isotropic panels because the assumption used in the analysis, that damping is independent of frequency, generally is invalid for isotropic materials. Since this assumption does not affect the results for free undamped vibration, the code also may be used to study the free undamped vibration of rectangular plates, cylinders, and cylindrical panels made of an isotropic material or alternating layers of isotropic and composite materials. In many cases, this code should prove to be a convenient means of designing such panels, even for those problems for which analyses have been reported previously but for which a supporting computer code is not readily available.

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